Many Muslims quote the following part of the verse to support the notion that Islam as a religion was completed after the advent of Prophet Muhammad (pbuh) and that God completed His favour / blessings (ni’mah) specifically at this point and exclusively for the Muslims.

005:003
"...This day I have perfected (Arabic: akmultu) your religion for you, completed My favour (Arabic: ni’mati) upon you, and have chosen for you Islam as your religion..."

This view of exclusivity of 'favour' / blessings (ni’mah) does not find any support in the Quran.

THE SAME FAVOUR WAS COMPLETED FOR PROPHET JOSEPH, THE FAMILY OF PROPHETS JACOB, ISAAC AND ABRAHAM (pbuh) AND THEY ALL FOLLOWED ISLAM AS A RELIGION

012:006
“Thus will your Lord choose you (Joseph) and teach you the interpretation of stories (and events) and complete His favour (Arabic: ni’matahu) on you and to the posterity of Jacob even as He completed it on your two forefathers Abraham and Isaac before! for God is full of knowledge and wisdom.”

As is noted from the above verse, God's favour / blessing (ni’mah) is not a specific reference to the dreams or interpretations of Prophet Joseph (pbuh) as a similar favour had been bestowed on the posterity of Jacob, as it was bestowed on both Prophets Abraham and Isaac. (pbuh)

Therefore, the 'favour' (ni’mah) must retain a wider meaning.
Prophet Muhammad (pbuh) and new Muslims:

005:003
"...and I have completed (Arabic: wa-atmamtu) upon you (Arabic: alaykum) my favour (Arabic: ni’mati)...."

Joseph, Jacob’s posterity, Abraham and Isaac; (pbut)

012:006
"...and complete (Arabic: wa-yutimmu) His favour (Arabic: ni-matahu) on you ...

WHAT WAS THE FAVOUR / BLESSING (NI’MAH)?

The first point to note in the context of the Quranic narrative is that 'Islam' never required perfection. Islam per se, has always remained wholly perfect and has been a religion preached by all Prophets.

042.013 (part)
"The same religion (Arabic: Deen) has He established for you as that which He enjoined on Noah and that which We have sent by inspiration to thee and that which We enjoined on Abraham, Moses, and Jesus ..."

The Arabs during Muhammad’s ministry no doubt also had a religion that they already observed and practiced. It is to be expected that certain parts of any religion may have overlaps with Islam. In this way, the religion of the ancient Arabs would have also been no different.

However, by ‘perfecting’ the imperfect religion of the Arab’s by a process of removing alien doctrines, blasphemous practices and those ways incongruent with Islam, their Lord purified their religion and perfected it by bringing it back to the ‘system’ he had enjoined on all believers before them (i.e. Islam).

Please see article (1) below

The Quran offered itself as a ‘furqan’ (25:1), a criterion that distinguished between right and wrong and a tool for discernment.

It is this assistive discerning guidance for believing servants which has always been promised (2:38, 20:123; 44:3-5) that constituted the favour (ni’mah).

020:123
“He said: Go down hence, both of you, one of you an enemy to the other. But when there comes to you from Me a guidance, then whosoever follows My guidance, he will not go astray nor come to grief”

Therefore, it is clear that that the completion of the favour and imparting guidance towards the religion of truth (Islam) was neither exclusive nor unique to Prophet Muhammad (pbuh) and his contemporaries. Rather, God had completed his favour on the Prophets and communities before Prophet Muhammad (pbuh) and had similarly chosen Islam for them as a religion.
FINAL THOUGHTS

Contrary to popular Muslim belief, neither is the religion of Islam new, nor did it initiate with the advent of Prophet Muhammad's (pbuh) ministry. Rather, it was a re-establishment of the same religion that had been inspired in all the Prophets before Prophet Muhammad (pbuh).

Similarly, God's favour was not completed exclusively for the Muslims of Muhammad's (pbuh) time but God had bestowed His favour on many Prophets before Prophet Muhammad (pbuh) and on their communities.

Related Article:

(1) What is the True Definition of 'Deen' from a Quran's Perspective?